International Journal of Engineering Research & Management Technology ISSN: 2348-4039
January-2016 Volume 3, Issue-1
Email: editor@ijermt.org www.ijermt.org

Recital Procedures for Summer Air Conditioner with Preemptive Repeat Repair

Ajay Kumar
Research Scholar
Swami Vivekanand University
Sagar, M.P.

ABSTRACT:
In this paper, the author has considered a summer air conditioning system for a place in hot and dry weather. The author has computed the availability and profit function for this system. Such systems are used for hot and dry outdoor conditions like Nagpur, Delhi, Bhopal and other place. The comfort conditions required in an air-conditioned space are 24^0C DBT (dry bulb temperature) and 60% RH (relative humidity). The whole system is divided into five subsystems namely A, B, C, D and E. These subsystems are air dampers, air filter, cooling coils, adiabatic humidifier and water eliminator, respectively. All these subsystems are connected in series. The whole system reaches to failed state on failure of any of its subsystems A, C, D and E. On the other hand, the whole system works in reduced efficiency on failure of subsystems B. The whole system can also be failed due to wear-out reasons.

KEY WORDS*:* summer, conditioning system

INTRODUCTION:
The whole system can also be failed due to wear-out reasons. Transition-state diagram for considered system has been shown in fig-1(b). All failures follow exponential time distribution whereas all repairs follow general time distribution. Pre-emptive repeat policy has been adopted for repair purpose. Asymptotic behaviour and a particular case, when repairs follow exponential time distribution, have been computed to enhance practical utility of the system. Reliability, availability and M.T.T.F. for considered system have been obtained.

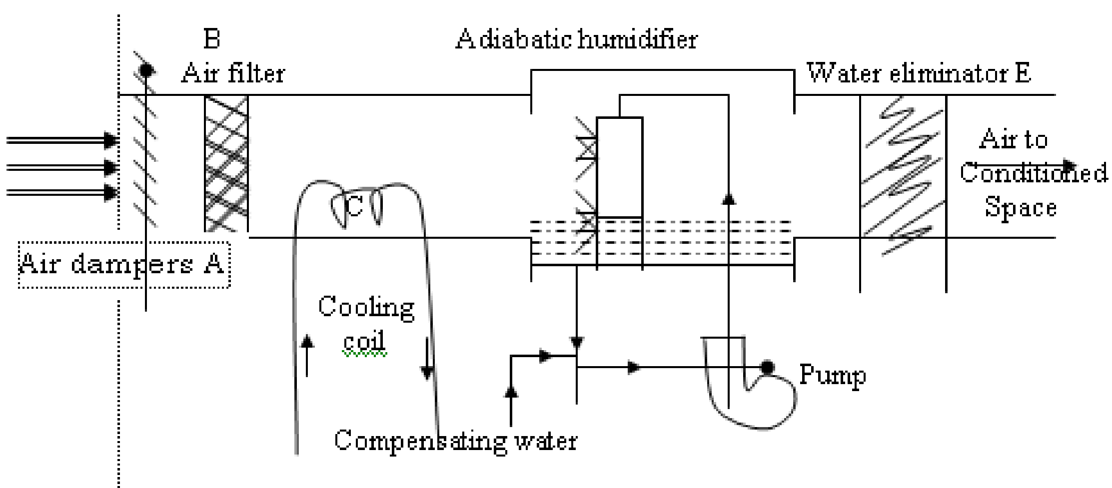

Fig-1(a): Summer Air conditioning system for hot and dry weather

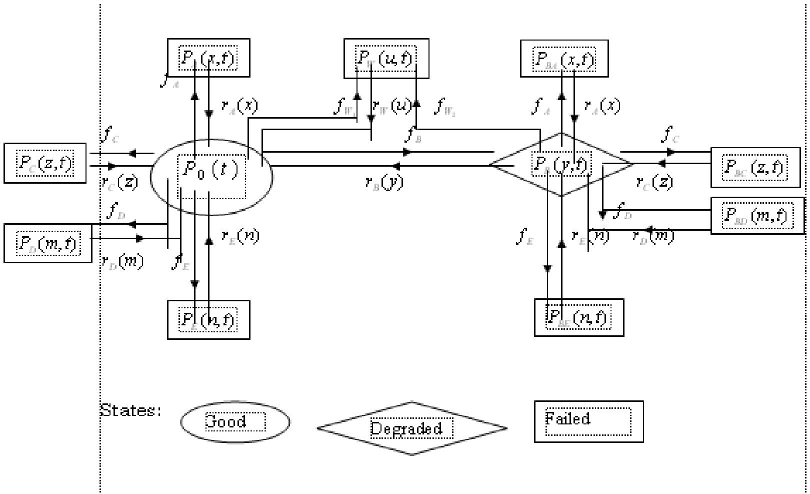

Fig- 1(b): State-transition diagram

FORMULATION OF MATHEMATICAL MODEL:

Using probability considerations and limiting procedure, we obtain the following set of difference-differential equations which is continuous in time, discrete in space and governing the behaviour of considered system:

$$\left[\frac{d}{dt}+f_A+f_B+f_C+f_D+f_E+f_{W_1}\right]P_0(t)=\int_0^\infty P_A(x,t)r_A(x)dx+\int_0^\infty P_B(y,t)r_B(y)dy$$
$$+\int_0^\infty P_C(z,t)r_C(z)dz+\int_0^\infty P_D(m,t)r_D(m)dm$$
$$+\int_0^\infty P_E(n,t)r_E(n)dn+\int_0^\infty P_W(u,t)r_W(u)du \qquad \ldots(1)$$

$$\left[\frac{\partial}{\partial j}+\frac{\partial}{\partial t}+r_i(j)\right]P_i(j,t)=0 \qquad \ldots(2)$$

where i = A,C,D,E and j = x,z,m,n respectively.

$$\left[\frac{\partial}{\partial u}+\frac{\partial}{\partial t}+r_W(u)\right]P_W(u,t)=0 \qquad \ldots(3)$$

$$\left[\frac{\partial}{\partial y}+\frac{\partial}{\partial t}+f_A+f_C+f_D+f_E+f_{W_2}+r_B(y)\right]P_B(y,t)=0 \qquad \ldots(4)$$

$$\left[\frac{\partial}{\partial j}+\frac{\partial}{\partial t}+r_i(j)\right]P_{Bi}(j,t)=0 \qquad \ldots(5)$$

where i = A,C,D,E and j = x,z,m,n respectively.

BOUNDARY CONDITIONS ARE:

$$P_i(0,t)=f_i\,P_0(t), \quad \text{where } i = \text{A,C,D and E} \qquad \ldots(6)$$

$$P_W(0,t)=f_{W_1}\,P_0(t)+f_{W_2}\,P_B(t) \qquad \ldots(7)$$

$$P_B(0,t)=\int_0^\infty P_{BA}(x,t)r_A(x)dx+\int_0^\infty P_{BC}(z,t)r_C(z)dz \qquad \ldots(8)$$

$$+\int_0^\infty P_{BD}(m,t)r_D(m)dm+\int_0^\infty P_{BE}(n,t)r_E(n)dn+f_BP_0(t)$$
…(9)

$$P_{Bi}(0,t)=f_i\,P_B(t) \qquad \text{where } i = \text{A,C,D and E}$$
…(10)

INITIAL CONDITIONS ARE:

$P_0(0)=1$, otherwise zero

SOLUTION OF THE MODEL:

In order to solve the above set of equations to obtain different state probabilities, taking Laplace transforms of equations (1) through (9) subjected to initial conditions (10), we get:

$$\left[s+f_A+f_B+f_C+f_D+f_E+f_{W_1}\right]\overline{P}_0(s)=1+\int_0^\infty \overline{P}_A(x,s)r_A(x)dx+\int_0^\infty \overline{P}_B(y,s)r_B(y)dy$$

$$+\int_0^\infty \overline{P}_C(z,s)r_C(z)dz+\int_0^\infty \overline{P}_D(m,s)r_D(m)dm$$

$$+\int_0^\infty \overline{P}_E(n,s)r_E(n)dn+\int_0^\infty \overline{P}_W(u,s)r_W(u)du \qquad \text{…(11)}$$

$$\left[\frac{\partial}{\partial j}+s+r_i(j)\right]\overline{P}_i(j,s)=0 \qquad \text{…(12)}$$

where i = A,C,D,E and j = x,z,m,n respectively.

$$\left[\frac{\partial}{\partial u}+s+r_W(u)\right]\overline{P}_W(u,s)=0 \qquad \text{…(13)}$$

$$\left[\frac{\partial}{\partial y}+s+f_A+f_C+f_D+f_E+f_{W_2}+r_B(y)\right]\overline{P}_B(y,s)=0 \qquad \text{…(14)}$$

$$\left[\frac{\partial}{\partial j}+s+r_i(j)\right]\overline{P}_{Bi}(j,s)=0 \qquad \text{…(15)}$$

where i = A,C,D,E and j = x,z,m,n respectively.

$$\overline{P}_i(0,s)=f_i\,\overline{P}_0(s), \qquad \text{where } i = \text{A,C,D and E} \qquad \text{…(16)}$$

$$\overline{P}_W(0,s)=f_{W_1}\,\overline{P}_0(s)+f_{W_2}\,\overline{P}_B(s) \qquad \text{…(17)}$$

$$\overline{P}_B(0,s)=f_B\overline{P}_0(s)+\int_0^\infty \overline{P}_{BA}(x,s)r_A(x)dx+\int_0^\infty \overline{P}_{BC}(z,s)r_C(z)dz$$
…(18)

$$+\int_0^\infty \overline{P}_{BD}(m,s)r_D(m)dm+\int_0^\infty \overline{P}_{BE}(n,s)r_E(n)dn$$

$$\overline{P}_{Bi}(0,s)=f_i\,\overline{P}_B(s) \qquad \text{where } i = \text{A,C,D and E} \qquad \text{…(19)}$$

Now integrate equation (12) by using boundary conditions (16), we have

$$\overline{P}_i(j,s)=f_i\,\overline{P}_0(s)\exp\left\{-sj-\int r_i(j)dj\right\}$$

integrating this again w.r.t. 'j' from 0 to ∞, we get

$$\bar{P}_i(s) = f_i\, \bar{P}_0(s) \frac{1-\bar{S}_i(s)}{s}$$

or, $\bar{P}_i(s) = f_i\, \bar{P}_0(s) D_i(s)$ for i = A,C,D and E ...(20)

Similarly, equation (13) gives on integration subjected to boundary condition (17):

$$\bar{P}_W(s) = \left[f_{W_1}\, \bar{P}_0(s) + f_{W_2}\, \bar{P}_B(s) \right] D_W(s) \qquad ...(21)$$

Integrate (15) by making use of (19), we obtain

$$\bar{P}_{Bi}(j,s) = f_i\, \bar{P}_B(s) \exp\left\{ -sj - \int r_i(j)\,dj \right\}$$

integrating this again w.r.t. j from 0 to ∞, we have

$\bar{P}_{Bi}(s) = f_i\, \bar{P}_B(s) D_i(s)$ for i = A,C,D and E ...(22)

Now simplifying (18) subjected to relevant relations, we get

$$\bar{P}_B(0,s) = f_B\, \bar{P}_0(s) + f_A\, \bar{P}_B(s)\bar{S}_A(s) + f_C\, \bar{P}_B(s)\bar{S}_C(s) + f_D\, \bar{P}_B(s)\bar{S}_D(s) + f_E\, \bar{P}_B(s)\bar{S}_E(s) \qquad ...(23)$$

Equation (14) gives on integration:

$$\bar{P}_B(y,s) = \bar{P}_B(0,s) \exp\left\{ -\left(s + f_A + f_C + f_D + f_E + f_{W_2}\right) y - \int r_B(y)\,dy \right\}$$

integrating this again w.r.t. y from 0 to ∞, we obtain

$$\bar{P}_B(s) = \bar{P}_B(0,s) D_B\left(s + f_A + f_C + f_D + f_E + f_{W_2}\right)$$

or, $\bar{P}_B(s) = \bar{P}_B(0,s) D_B(N)$

where, $N = s + f_A + f_C + f_D + f_E + f_{W_2}$

using (23), it gives

$$\bar{P}_B(s)\left[1 - \left\{ f_A\, \bar{S}_A(s) + f_C\, \bar{S}_C(s) + f_D\, \bar{S}_D(s) + f_E\, \bar{S}_E(s) \right\} D_B(N) \right] = f_B \bar{P}_0(s) D_B(N)$$

$$\therefore \bar{P}_B(s) = \frac{f_B D_B(N) \bar{P}_0(s)}{1 - \left\{ f_A\, \bar{S}_A(s) + f_C\, \bar{S}_C(s) + f_D\, \bar{S}_D(s) + f_E\, \bar{S}_E(s) \right\} D_B(N)} \qquad ...(24)$$

or, $\bar{P}_B(s) = A(s)\bar{P}_0(s)$ (Say)

Finally, simplifying equation (11) with the help of relevant expressions, we have

$$\bar{P}_0(s) = \frac{1}{C(s)}$$

Thus, we obtain the following L.T. of various state probabilities, depicted in fig-1(b), in terms of C(s):

$$\bar{P}_0(s) = \frac{1}{C(s)} \qquad ...(25)$$

$$\bar{P}_i(s) = \frac{f_i\, D_i(s)}{C(s)} \quad , \quad i = \text{A,C,D and E} \qquad ...(26)$$

$$\bar{P}_W(s) = \frac{1}{C(s)} \left[f_{W_1} + f_{W_2}\, A(s) \right] D_W(s) \qquad ...(27)$$

$$\bar{P}_B(s) = \frac{A(s)}{C(s)} \qquad ...(28)$$

$$\bar{P}_{Bi}(s) = \frac{f_i\, A(s) D_i(s)}{C(s)} \quad , \quad i = \text{A,C,D and E} \qquad ...(29)$$

where, $A(s) = \dfrac{f_B D_B(N)}{1 - \left\{ f_A\, \bar{S}_A(s) + f_C\, \bar{S}_C(s) + f_D\, \bar{S}_D(s) + f_E\, \bar{S}_E(s) \right\} D_B(N)}$ ...(30)

$$N = s + f_A + f_C + f_D + f_E + f_{W_2} \qquad \ldots(31)$$

and $$C(s) = s + f_A + f_B + f_C + f_D + f_E + f_{W_1} - f_A \overline{S}_A(s) - f_C \overline{S}_C(s) - f_D \overline{S}_D(s) - f_E \overline{S}_E(s) - \left[f_{W_1} + f_{W_2} A(s)\right]\overline{S}_W(s) - \left[f_B + \left\{f_A \overline{S}_A(s) + f_C \overline{S}_C(s) + f_D \overline{S}_D(s) + f_E \overline{S}_E(s)\right\}A(s)\right]\overline{S}_B(N) \qquad \ldots(32)$$

VERIFICATION:

It is interesting to note here that

sum of equations (25) through (29) $= \dfrac{1}{s}$ …(33)

STEADY-STATE BEHAVIOUR OF THE SYSTEM:

Using final value theorem in L.T., *viz*, $\lim_{t\to\infty} P(t) = \lim_{s\to 0} s\overline{P}(s) = P(say)$, provided the limit on LHS exists, in equations (25) through (29), we obtain the following steady-state behaviour of the considered system:

$$P_0 = \frac{1}{C'(0)} \qquad \ldots(34)$$

$$P_i = \frac{f_i M_i}{C'(0)} \quad , \quad i = \text{A,C,D and E} \qquad \ldots(35)$$

$$P_W = \frac{1}{C'(0)}\left[f_{W_1} + f_{W_2} A(0)\right]M_W \qquad \ldots(36)$$

$$P_B = \frac{A(0)}{C'(0)} \qquad \ldots(37)$$

$$P_{Bi} = \frac{f_i A(0) M_i}{C'(0)} \quad , \quad i = \text{A,C,D and E} \qquad \ldots(38)$$

where, $C'(0) = \left[\dfrac{d}{ds} C(s)\right]_{s=0}$

$M_i = -\overline{S}_i{}'(0) =$ mean time to repair i^{th} failure.

and $A(0) = \dfrac{f_B D_B(N-s)}{1-(f_A + f_C + f_D + f_E)D_B(N-s)}$

PARTICULAR CASE:

WHEN REPAIRS FOLLOW EXPONENTIAL TIME DISTRIBUTION:

In this case, setting $\overline{S}_i(j) = \dfrac{r_i}{j + r_i}, \forall\ i \text{ and } j$ in equations (25) through (29) we obtain the following L.T. of various states probabilities of fig-1(b):

$$\overline{P}_0(s) = \frac{1}{E(s)} \qquad \ldots(39)$$

$$\overline{P}_i(s) = \frac{f_i}{E(s)(s + r_i)} \quad , \quad i = \text{A,C,D and E} \qquad \ldots(40)$$

$$\overline{P}_W(s) = \frac{1}{E(s)}\left[f_{W_1} + f_{W_2} Q(s)\right]\frac{1}{s + r_W} \qquad \ldots(41)$$

$$\overline{P}_B(s) = \frac{Q(s)}{E(s)} \qquad \ldots(42)$$

$$\overline{P}_{Bi}(s) = \frac{f_i\,Q(s)}{E(s)(s+r_i)} \quad , \qquad i = \text{A,C,D and E} \qquad \ldots(43)$$

where, $$Q(s) = \frac{f_B\left[1-\overline{S}_B(N)\right]}{N - \left(\frac{f_A r_A}{s+r_A} + \frac{f_C r_C}{s+r_C} + \frac{f_D r_D}{s+r_D} + \frac{f_E r_E}{s+r_E}\right)\left[1-\overline{S}_B(N)\right]} \qquad \ldots(44)$$

and $$E(s) = s + f_A + f_B + f_C + f_D + f_E + f_{W_1} - \frac{f_A r_A}{s+r_A} - \frac{f_C r_C}{s+r_C} - \frac{f_D r_D}{s+r_D} - \frac{f_E r_E}{s+r_E}$$
$$-\left[f_{W_1} + f_{W_2}Q(s)\right]\frac{r_W}{s+r_W}$$
$$-\left[f_B + \left\{\frac{f_A r_A}{s+r_A} + \frac{f_C r_C}{s+r_C} + \frac{f_D r_D}{s+r_D} + \frac{f_E r_E}{s+r_E}\right\}Q(s)\right]\frac{r_B}{N+r_B} \qquad \ldots(45)$$

N has been mentioned earlier in equation (31).

RELIABILITY AND M.T.T.F. OF THE SYSTEM:

We have from equation (25)

$$\overline{R}(s) = \frac{1}{s + f_A + f_B + f_C + f_D + f_E + f_{W_1}}$$

Taking inverse L.T., we get

$$R(t) = \exp\left\{-\left(f_A + f_B + f_C + f_D + f_E + f_{W_1}\right) t\right\} \qquad \ldots(46)$$

Also, $$M.T.T.F. = \int_0^\infty R(t)dt$$

$$= \frac{1}{f_A + f_B + f_C + f_D + f_E + f_{W_1}} \qquad \ldots(47)$$

Table-1

t	R(t)
0	1
1	0.892258
2	0.796124
3	0.710348
4	0.633814
5	0.565525
6	0.504595
7	0.450229
8	0.40172
9	0.358438
10	0.319819

Table-2

t	$P_{up}(t)$
0	1
1	0.894039
2	0.799296
3	0.714585
4	0.638844
5	0.571125
6	0.510578
7	0.456444
8	0.408046
9	0.364775
10	0.326089

Table-3

f_B	M.T.T.F.
0	8.928571
0.001	8.849558
0.002	8.77193
0.003	8.695652
0.004	8.62069
0.005	8.547009
0.006	8.474576
0.007	8.403361
0.008	8.333333
0.009	8.264463
0.01	8.196721

RESULTS AND DISCUSSION:

Table-1 gives the values of reliability of considered system for various values of time *t.* Analysis of table-1 reveal that the reliability of considered system decreases approximately in constant manner and there are no sudden jumps in the values of reliability.

Table-2 gives the values of availability of considered system for different values of time *t*. Critical examination of table-2 yield that value of availability decreases rapidly in the beginning but thereafter it decreases constantly.

Table-3 gives the values of M.T.T.F. of considered system for different values of failure rate of subsystem *B*. Analysis of table-3 yield that value of M.T.T.F. decreases catastrophically.

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